Non linear image filtering: a PDE approach

Ph. Bolon

LISTIC / Polytech Annecy-Chambéry
University of Savoie

OUTLINES

1- Introduction and example

2- Diffusion Filters

3- Shock Filters

4- Morphological Filters

5- Conclusions

algorithmic point of view
extensions: multivariate, curvature, ...

INTRODUCTION

principle

local operators

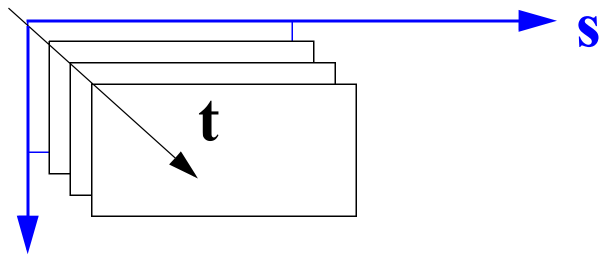
iterative processing

notations

s : site (=pixel)

t : time (= iteration number)

$x(s,t)$; $x(s,0) = x_0(s)$



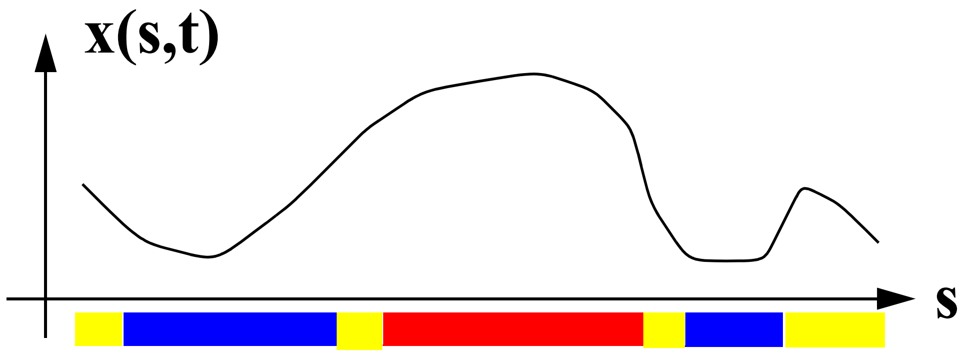
processing model

$$\frac{\partial x}{\partial t} = F\left(x, \frac{\partial}{\partial s}, \frac{\partial^2}{\partial s^2}\right)$$ gradient, laplacian,

historical example

- $$\frac{\partial x}{\partial t} = c\frac{\partial^2 x}{\partial s^2}$$

heat transfer equation => Fourier analysis



operator: smoothing effect

solution: solving a partial differential equation
calculus
computation

assume: for all t, x(s,t) is L_2

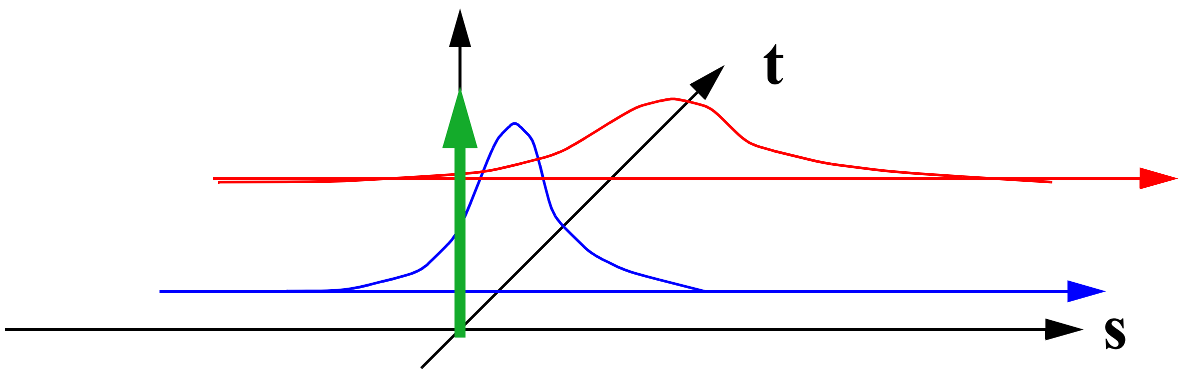
$$\tilde{x}(u,t) = FT\{x(s,t)\}$$

$$\tilde{x}(u,t) = X(u)e^{-4\pi^2 ctu^2}$$

$$x(s,t) = x_0(s) * g(s,t)$$

avec

$$g(s,t) = \frac{1}{\Delta\sqrt{2\pi}}\exp\left(-\frac{s^2}{2\Delta^2}\right) \text{ et } \Delta = \sqrt{8\pi ct}$$

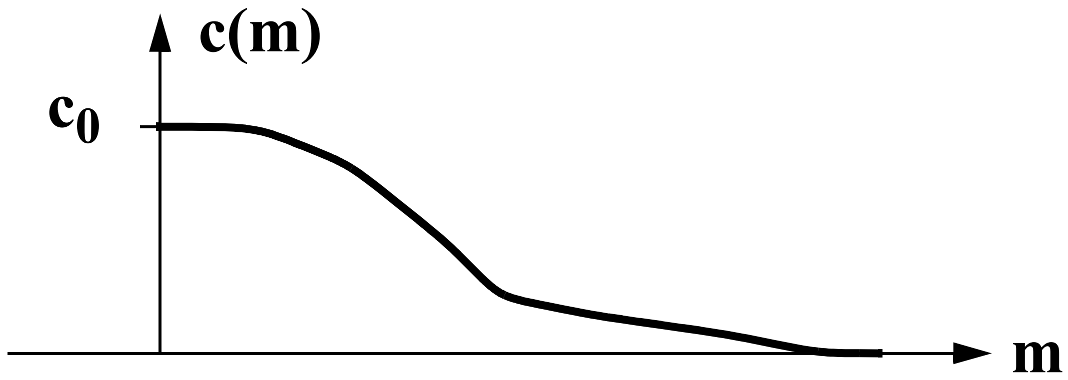


demo

DIFFUSION FILTERS

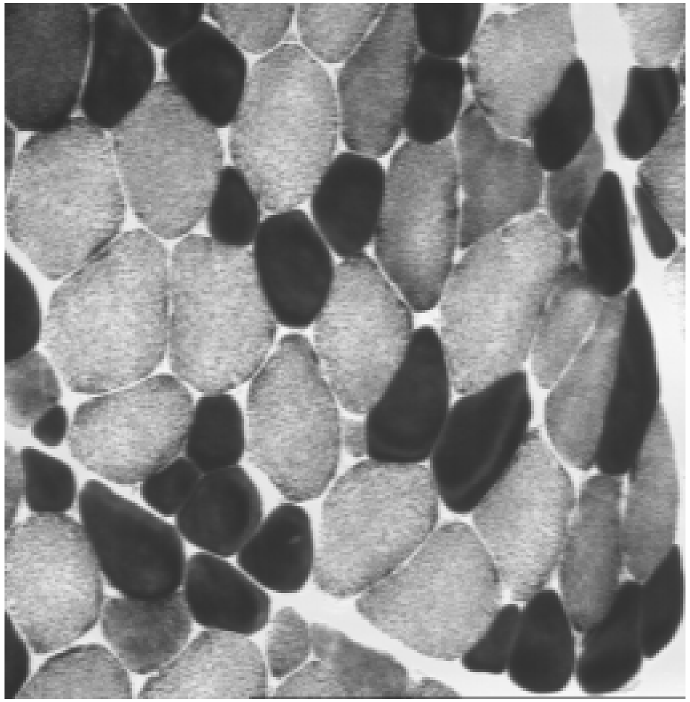
- $$\frac{\partial x}{\partial t} = div(c(|\nabla x|)\nabla x)$$

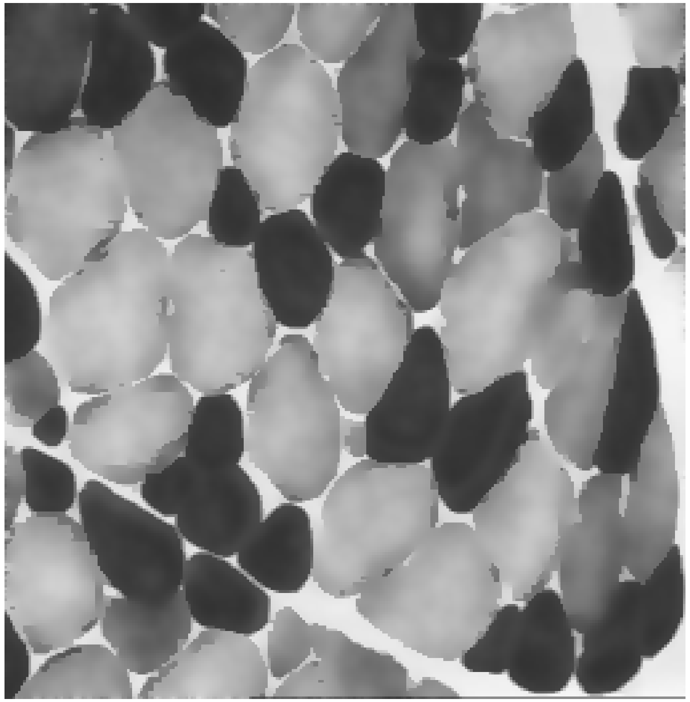
c : decreasing function



edge: identity (narrow convolution kernel)
noisy region: smoothing (large kernel)

anisotropic diffusion

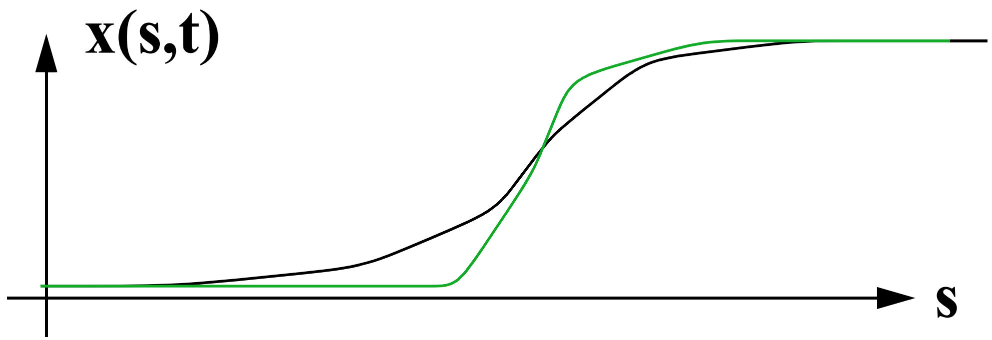




4-neighborhood
k = 10
50 itérations
Δt = 0.2

SHOCK FILTERS

- $$\frac{\partial x}{\partial t} = -c\frac{\partial^2 x}{\partial s^2}$$

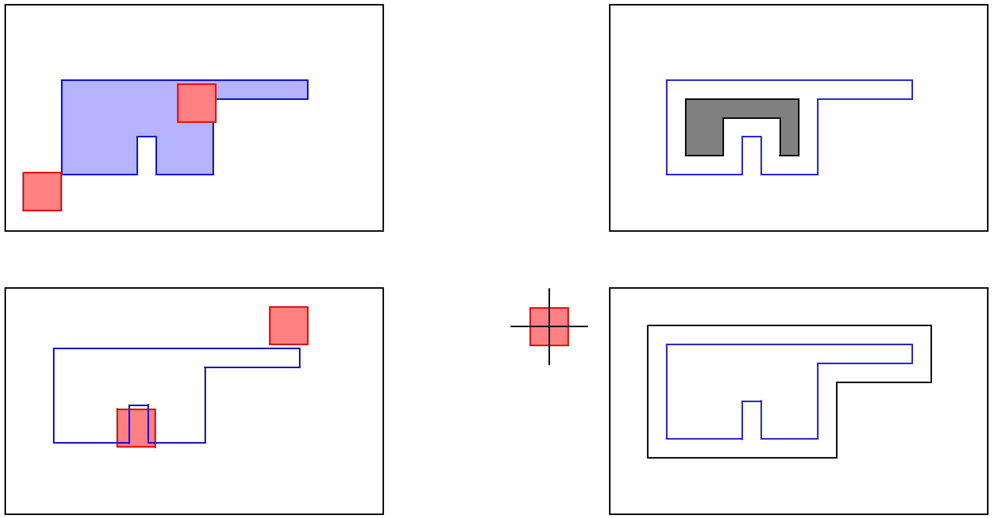


unsharp masking

MORPHOLOGICAL OPERATORS

- $$\frac{\partial x}{\partial t} = c|\nabla x|$$ **c = +/- 1**

dilation
erosion



CONCLUSIONS

- **algorithms**
  - **local operators**
  - **requires only a few neighboring pixels**

- **extensions**
  - **3D**
  - **curvature**
  - **color / multivariate images**