

# Comparison of gait variability and stability indices

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**Abstract**—Indices of variability and stability are essential in evaluating the motions of the human gait. Owing to the inconsistent results of earlier studies, the relationships among popular indices, i.e., maximum Lyapunov exponent ( $\lambda$ ), MeanSD, and long-range correlation ( $\alpha$ ), are inconclusive. In this study, we examined the correlations among these indices using the velocity of the center of body mass for 10 participants walking on a treadmill. In terms of the velocity along the gravitational direction, MeanSD exhibited a negative correlation with the short-term  $\lambda_S$  and the long-term  $\lambda_L$  when the walking speed was 4.0 km/h. Although a positive correlation between MeanSD and  $\lambda$  is expected based on the meanings of these indices, the present and earlier studies collectively suggest that there is no positive correlation between MeanSD and  $\lambda$ . In the same direction, a positive correlation was found between  $\lambda_S$  and  $\alpha$  when the walking speed was 3.5 km/h. MeanSD and  $\alpha$  tended to exhibit a negative correlation for some conditions, which is reasonable because MeanSD and  $\alpha$  indicate the level of variability and the long-term consistency of walking, respectively.

**Index Terms**—gait variability, human locomotion, long-range correlation, maximum Lyapunov exponent, MeanSD

## I. INTRODUCTION

To create systems that assist human locomotion, it is essential to quantify the variability and stability of walking. Many indices have been proposed and compared thus far.

MeanSD is a popular index for gait variability [1], [2]. It is the mean of the standard deviation of walking parameters. Another index is the long-range correlation ( $\alpha$ ). It evaluates the system's error-tolerance and resistance to external perturbation and quantifies the long-term consistency of the system. The maximum Lyapunov exponent ( $\lambda$ ) is an index for gait stability. It quantifies the reaction of the system against an infinitesimal perturbation [1], [3].

Although the relationship between  $\lambda$  and MeanSD has been discussed in the past, their relationship remains unclear. Dingwell et al. [3] did not find significant correlations between  $\lambda$  and MeanSD. Terrier et al. [4] reported a negative correlation between the long-term maximum Lyapunov exponent ( $\lambda_L$ ) and MeanSD for gait motions on a treadmill. Brujin et al. [1] reported a positive correlation between the short-term maximum Lyapunov exponent ( $\lambda_S$ ) and MeanSD. These studies are inconsistent in terms of the relationships between  $\lambda$  and MeanSD. For  $\alpha$  and  $\lambda$ , Terrier et al. [4] reported that there was no correlation between them.

In this study, we examined and discussed the correlations among maximum Lyapunov exponent, MeanSD, and long-range correlation using the data of 10 healthy males walking on a treadmill [5].

## II. COMPUTATION OF GAIT INDICES

### A. Maximum Lyapunov exponent

The maximum Lyapunov exponent was computed as follows [3], [4], [6]. State vector  $\mathbf{S}(t)$  was constructed from discrete time series  $q(t)$ , which was the velocity of the center of body mass in the present study, as  $\mathbf{S}(t) = [q(t), q(t + \tau), \dots, q(t + (d - 1)\tau)]$  where  $\tau$  is the time delay and  $d$  is the dimension of  $\mathbf{S}(t)$ .  $\mathbf{S}(t)$  forms a limit cycle in the state space over  $t$ . Euclidean distances between neighboring trajectories of the limit cycle were calculated. Subsequently, they were averaged over all discrete time points to obtain the average logarithmic rate of divergence. The slope of the average logarithmic rate of divergence is represented by  $\lambda$ . In this study, we computed the slope using the first 0.5–1 stride for the short-term ( $\lambda_S$ ) and fourth to tenth strides for the long-term ( $\lambda_L$ ). We set  $\tau$  to be 10% gait cycle and  $d = 5$ .

### B. Long-range correlation

The long-range correlation was computed as follows [4], [7]. The original time series was first integrated to be another time series  $y(k)$  of length  $N$  as described in [4] and [7]. Subsequently,  $y(k)$  was divided into boxes with equal length of  $n$ . In each box, a regression line  $y_n(k)$  was fitted to  $y(k)$ .  $y(k)$  was then detrended by subtracting the local trend  $y_n(k)$  in each box. The average fluctuation  $F(n)$  for box size  $n$  was calculated by

$$F(n) = \sqrt{\frac{1}{N} \sum_{k=1}^N [y(k) - y_n(k)]^2} \quad (1)$$

The gradient of  $\log F(n)$  to  $\log(n)$  was defined as the long-range correlation ( $\alpha$ ).  $\alpha$  usually fits into 0.5–1.0, and a greater  $\alpha$  value indicates more self-correlated walking. We varied the box size  $n$  from 100 to 1000.

### C. MeanSD

MeanSD is an index of gait variability. To compute MeanSD, the periods of walking were first divided into 100% gait cycles. For each percentage, the standard deviation of the gait parameter, i.e., the velocity of the center of body mass, was calculated. The mean of the standard deviations over all percentages is the MeanSD [3].

## III. WALKING MOTION DATA

Ten participants' walking motions on a treadmill were recorded at 100 Hz at both 3.5 km/h and 4.0 km/h [5]. For each participant, we calculated the indices of Sec. II using the velocities of the center of body mass. We excluded the first ten

TABLE I: Correlation coefficients between MeanSD, maximum Lyapunov exponent ( $\lambda_L$  and  $\lambda_S$ ), and long-range correlation ( $\alpha$ ). \* and + indicate the significance levels of  $p = 0.05$  and  $p = 0.10$ , respectively.

	3.5 km/h			4.0 km/h			Overall		
	x	y	z	x	y	z	x	y	z
MeanSD and $\alpha$	0.09	-0.40 <sup>+</sup>	-0.36	-0.59 <sup>+</sup>	-0.44	-0.03	-0.27	-0.40	-0.22
MeanSD and $\lambda_S$	-0.25	-0.32	-0.16	0.20	-0.18	-0.64*	-0.10	-0.21	-0.21
MeanSD and $\lambda_L$	0.32	0.30	0.09	0.11	-0.42	-0.72*	0.24	-0.1	-0.40 <sup>+</sup>
$\lambda_S$ and $\alpha$	0.03	0.14	0.75*	0.10	0.37	-0.34	0.06	0.32	0.37
$\lambda_L$ and $\alpha$	0.11	-0.35	0.33	0.42	0.40	-0.07	0.30	0.24	0.09



Fig. 1: Walking on treadmill

strides and selected 11th to 50th strides from the recorded data to compute the walking indices. One gait cycle was defined from the left heel contact (0%) to the next left heel contact (100%). The coordinate system was defined as follows: right as the x-axis, forward as the y-axis, and up as the z-axis as shown in Fig.1.

#### IV. RESULTS

Table I shows the correlation coefficients among the gait indices. The correlation coefficients were calculated by the walking speeds, i.e., 3.5 or 4.0 km/h. Further, the values shown in the three right columns were computed by collectively using the two walking speed conditions. Most values were not statistically significant, in part because of the small number of participants. For 4.0 km/h along the z-axis, we found a negative correlation ( $r = -0.64$ ,  $p = 0.047$ ) between MeanSD and  $\lambda_S$  and a negative correlation ( $r = -0.72$ ,  $p = 0.020$ ) between MeanSD and  $\lambda_L$ . For 3.5 km/h along the z-axis, we found a positive correlation ( $r = 0.75$ ,  $p = 0.012$ ) between  $\lambda_S$  and  $\alpha$ . In addition, we observed that MeanSD and  $\alpha$  tended to exhibit a negative correlation.

#### V. DISCUSSION

MeanSD is an index of the gait variability, and  $\lambda_S$  is an index of the instability at the beginning of walking. Thus, it is unintuitive that MeanSD and  $\lambda_S$  exhibit a negative correlation. However, in earlier studies, Dingwell et al. [3] and Terrier et al. [4] reported the same results (the overall correlation was weak and negative). A positive correlation between  $\lambda_L$ , which accumulates relatively long-term information of walking, and MeanSD, which calculates the variability of the entire walk, seems natural. However, such positive relationships have not been reported so far [1], [3], [4]. In our study, a weak negative correlation between them was observed. The present and related studies suggest that there is no positive correlation between  $\lambda$  and MeanSD. Note that there is no general explanation for the relationships between these two popular gait indices.

A negative correlation between MeanSD and  $\alpha$  is understandable, because MeanSD indicates the variability and  $\alpha$  indicates the long-term consistency of walking. As the variability increases (MeanSD increases), the long-range correlation decreases ( $\alpha$  decreases). This relationship was also reported in [4].

A positive correlation was found between  $\alpha$  and  $\lambda_S$  for 3.5 km/h along the z-axis. In an earlier study, Terrier et al. [4] stated “they seem not correlated together.” The computation of  $\alpha$  largely depends on the box size,  $n$ , and we are unable to determine the relationship between  $\alpha$  and  $\lambda$  without further research.

#### VI. CONCLUSION

We examined correlations among the gait variability and stability indices for treadmill walking, for which there has been no agreement thus far. Unexpectedly, the maximum Lyapunov exponent  $\lambda$  and MeanSD exhibited negative or insignificant correlations. The present and earlier studies [3], [4] suggest at least that there is no positive correlation between them. Furthermore, for a limited condition, a positive correlation was observed between  $\lambda_S$  and  $\alpha$ . MeanSD and  $\alpha$  tended to exhibit negative correlations.

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