Perceived Hardness through Actual and Virtual Damped Natural Vibrations

Kosuke Higashi, Shogo Okamoto, and Yoji Yamada

Abstract—Humans perceive hardness of objects from damped natural vibrations by tapping their surfaces. We investigated the roles of six parameters characterizing the vibration in virtual and real environments. The effects of some parameters differed between the two environments. This finding provides valuable information to the developers of haptic applications.

Index Terms—hardness, perception, tapping, vibration, virtual objects, real objects.

1 Introduction

HUMANS tap the surface of a rigid object to judge its hardness, whereas they press or push a deformable one for the same purpose. In the former situation, the vibrotactile signals caused by tapping are considered to be major perceptual cues [1], [2]. Although the mechanism of hardness perception by tapping has not been intensively studied thus far, a number of current haptic rendering algorithms and applications have utilized this perceptual phenomenon, because simulated vibrotactile stimuli can be enhancements or alternatives to force displays to present collisions with virtual objects [1], [2], [3], [4], [5], [6], [7]. In this study, we investigated vibration-based hardness perception by tapping while restricting or nullifying the effect of a static reaction force from objects, with the ultimate goal of devising better rendering algorithms and elucidating the perceptual mechanisms. In particular, we pursued the areas of consistency and inconsistency between the hardness perception experienced by tapping actual objects versus simulated vibrotactile stimuli. The results guide the effective usage of vibrotactile stimuli and raise problems with rendering methods which remained unsolved.

In general, the vibratory response of an object to an impulsive force input, such as a tap, is characterized by its amplitude, frequency, and attenuation. Among them, the vibratory frequency is known to affect the hardness perceived through the vibration; this is the most reliable property, and has been repeatedly proven in many studies. Higher frequency leads to the perception of greater hardness [1], [6], [8]. Because the natural frequency of the vibration is largely influenced by the stiffness of the object, the stiffness also influences not only the hardness perception by pressing but also by tapping. Furthermore, vibrations including multiple frequency components are felt to be more realistic and harder than those constituted by a single frequency component [2], [9].

In contrast, the perceptual effects of the amplitude and attenuation of the vibration have yet to be determined. Using a model based on momentum conservation, described below, the vibratory amplitude is determined by the term related to the contact velocity of the tapping finger and the term dependent on the object stiffness. As the stiffness increases, the amplitude becomes smaller. However, in perceptual experiments using force displays, the harder objects were either linked with the larger amplitudes [10], [11], or the effect of the amplitude was minor [6].

Few studies have investigated the perceptual effects of the damping properties of the vibrotactile stimuli, except for those on which the present study is based [8], [11]. The effects of the damping properties are still inconclusive. In the presentation of simulated vibrotactile stimuli using a commercially available force display, the effects of the viscosity of the virtual object, which largely determines the damping property of the vibration, depended on individuals [11]. For some participants, the viscosity exhibited a positive correlation with the perceived hardness; however, for others the viscosity exhibited either a negative correlation or an insignificant relationship with the perceived hardness. In tapping actual objects, the viscosity coefficients were positively correlated with perceived hardness [8]. Evidently,the results of these two studies on real and simulated tapping events are inconsistent.

In the present study, we investigated the perceptual effects of three vibration properties, including the amplitude, time constant, and frequency of the vibratory stimulus, and three mechanical parameters including the mass, viscosity, and stiffness of the object to be tapped. Both the real object samples and simulated vibratory stimuli were tested to investigate the consistency between the real and simulated stimuli. These samples were tested such that only the vibrotactile cues affected judgment of hardness, and effects of the static reaction force were nullified. The ranges of the mechanical parameters of the virtual and real stimuli partly overlapped, which enabled us to compare the results from each type of stimuli. All of the experimental procedures were approved by the internal review board of the School of Engineering in Nagoya University (#15-12).

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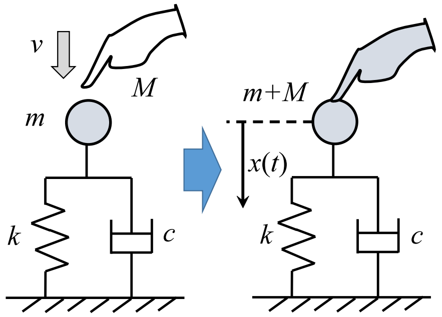

Fig. 1. Tapping of the object modeled as a one degree-of-freedom spring-mass-damper system.

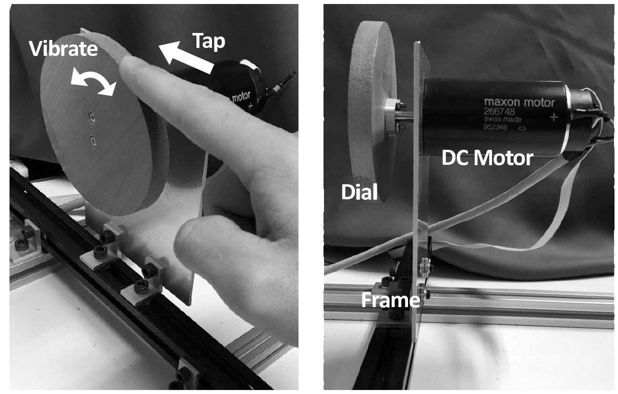

Fig. 2. One degree-of-freedom force display with direct DC motor drive.

2 Experiment 1: Subjective hardness of virtual objects

2.1 Modeling of tapping an object

We modeled the damped natural vibration as the impulse response of the one degree-of-freedom spring-mass-damper system shown in Fig. 1. Note that we did not reproduce the quasi-static force based on Hooke's law in this model. The hand with equivalent mass M collides with the object characterized by mass m, viscosity c, and stiffness k at a velocity v, and the impulse response is caused by the momentum of the contact object Mv. The mass and hand then collectively vibrate as one object with mass $m'(= m + M)$. The displacement of the vibration $x(t)$ at time t, with $t = 0$ being the moment of impact, is represented by

$$x(t) = A \exp \frac{-t}{\tau} \sin \omega t \tag{1}$$

$$A = A'v = \frac{2Mv}{\sqrt{k(4m'k - c^2)}} \tag{2}$$

$$\tau = \frac{2m'}{c} \tag{3}$$

$$\omega = 2\pi f = \frac{\sqrt{4m'k - c^2}}{2m'} \tag{4}$$

where A, A', τ, ω, and f are the amplitude, amplitude per contact speed, time constant, angular frequency, and frequency, respectively. The amplitude of the resultant vibration A is proportional to the contact speed of the hand, and can be decomposed into A' and v, where A' is fully determined by the mechanical parameters of the object and the hand mass. Hence, the vibration amplitude A was determined by both A' and v in the experiment.

Although the models for vibration feedback proposed in previous research slightly differ from each other, our vibration model reflects their physical soundness. In our model, the vibration amplitude is proportional to the contact speed, which is consistent with the previous models [1], [2], [7], [10]. The natural frequency increases with the increase in the stiffness of the object, which is true in our and former models [2], [10]. We also introduced the inverse proportional relationship between the time constant and the viscosity of the object [10] into our vibration model.

2.2 Force display

We assembled a haptic display, shown in Fig. 2. The main components of this apparatus are a dial, DC motor (RE40, Maxon, Switzerland), rotary encoder (Encoder MR Type L, Maxon, Switzerland), current controller (ADS50/10, Maxon, Switzerland), and microcomputer (mbed LPC1768, ARM, United Kingdom). The participant was able to experience a vibration stimulus by moving their fingertip forward to rotate the dial to a certain angle, which was measured by a rotary encoder installed on the DC motor. The output shaft of the motor was directly connected to the dial with no gear such that the rotation resistance remained very low when freely rotating. The control system employed a feedforward control method operated at 5 kHz. In order to reduce the risk of malfunction when the movable part was rotated at high speed, we adopted a dial, rather than an arm-shaped manipulator. The vibration was presented in the tangential direction of the dial. The tangential displacement of the dial can be described as $x(t) = r\theta(t)$ where r and θ are the radius of the dial and its rotational angle, respectively. We presented the various stimuli by changing the mass m, viscosity c, and stiffness k of the virtual object, and the amplitude per tapping speed A', time constant τ, and frequency f of the damped vibration.

In order to design the open loop control system, we identified the characteristics of the display system, including the tapping fingertip. We regarded the user's finger as the load mass of the motor actuation and its mechanical model as the one degree-of-freedom spring-mass-damper system. The output torque T necessary for actuating the dial at acceleration $\ddot{x}(t)$ is given by

$$T = rF(t) \tag{5}$$

$$F(t) = M\ddot{x}(t) + C\dot{x}(t) + Kx(t) \tag{6}$$

where r, F, M, C, and K are the radius of the dial, tangential force, equivalent mass, viscosity, and stiffness of the display system plus fingertip, respectively. We derived the transfer function between $F(t)$ and $x(t)$ by a vibrating test within the frequency range used in the experiment, in which the participants were three right-handed males in their twenties. Each placed an index finger on the edge of the dial with a relaxed posture, placing their elbow on an armrest. M, C, and K values were then estimated from the transfer function. The resulting mean values of M, C, and K were 0.02 kg, 4.5 Ns/m, and $1,000$ N/m, respectively. The values of the viscosity and stiffness might have been mostly from the finger skin because they were close to those

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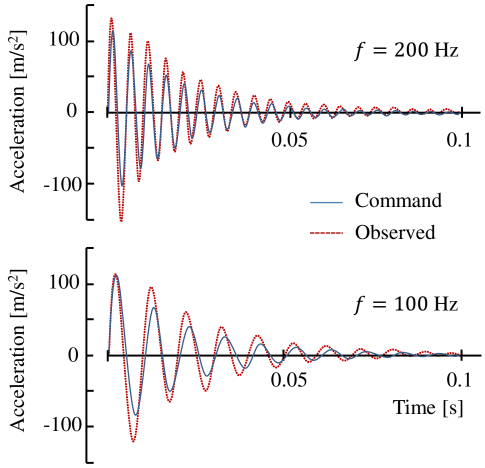

Fig. 3. Examples of vibratory stimuli produced by the force display with a fingertip on the dial at 200 Hz (top) and 100 Hz (bottom). The time constant was $\tau = 0.02$ s. The peak acceleration value was set to 120 m/s^2.

of the shear direction of fingertip skin reported in a past study [12].

Fig. 3 shows the examples of vibratory stimuli produced by the force display with a fingertip on the dial at 100 and 200 Hz, which was the maximum frequency used in the experiment.

2.3 Stimuli

We prepared three stimulus groups α, β, and γ. In group α, the vibration parameters (A', τ, and f) were independently varied, whereas the mechanical parameters (m, c, k) were independently varied in group β and γ. Such stimulus sets are ideal to conduct multiple regression analysis where explanatory variables are preferably independent. The stimuli in groups α and β included vibration only, whereas those in group γ included vibration and a virtual wall that caused a reaction force depending on the indentation depth. The stiffness of the wall was fixed to 5 N/m for all stimuli in group γ. This group allowed us to investigate the effects of vibration cue when the vibration and reaction force cues coexist.

The A', τ, and f values were independently varied in group α as listed in Table 1. The values of the reference stimulus, which was the center point for the variation of the stimuli, were set to $A' = 0.13 \times 10^{-3}$ s, $\tau = 25 \times 10^{-3}$ s, and $f = 100$ Hz. We prepared thirteen test stimuli, including twelve stimuli with one of their vibration properties changed from the reference values, and the reference stimulus itself. The amplitude per contact speed A' and frequency f of the test stimuli varied by factors of $1/2$, $1/\sqrt{2}$, $\sqrt{2}$, and 2 times those of the reference stimulus. The time constant τ of the test stimuli was quartered, halved, doubled, and quadrupled from that of the reference stimulus.

Group β consisted of stimuli with different mass m, viscosity c, and stiffness k as listed in Table 2. The physical properties of the reference stimulus were set to $m = 0.1$ kg, $c = 10$ Ns/m, and $k = 50,000$ N/m. We prepared thirteen test stimuli, including twelve stimuli with one of their physical properties changed from the reference values, and the reference stimulus itself. The physical properties of the test stimuli were quartered, halved, doubled, and quadrupled from the reference values. These three parameters were independently varied. More than two vibration properties were changed when a change was made in either m, c, or k, as expressed by (2), (3), and (4).

Group γ included the same stimuli as group β; however, those in group γ were presented with a virtual wall with a spring coefficient of 5 N/m.

TABLE 1
Stimuli used in Experiment 1α

	A' [ms]	τ [ms]	f [Hz]
Reference Stimulus	0.13	25	100
Test Stimulus	0.13	25	100
	0.065	25	100
	0.092	25	100
	0.18	25	100
	0.26	25	100
	0.13	100	100
	0.13	50	100
	0.13	12.5	100
	0.13	6.25	100
	0.13	25	50
	0.13	25	71
	0.13	25	141
	0.13	25	200

TABLE 2
Stimuli used in Experiment 1β and 1γ

	m [kg]	c [Ns/m]	k [N/m]
Reference Stimulus	0.1	10	50000
Test Stimulus	0.1	10	50000
	0.025	10	50000
	0.05	10	50000
	0.2	10	50000
	0.4	10	50000
	0.1	2.5	50000
	0.1	5	50000
	0.1	20	50000
	0.1	40	50000
	0.1	10	12500
	0.1	10	25000
	0.1	10	100000
	0.1	10	200000

2.4 Tasks

Participants evaluated the subjective hardness of the test stimulus based on the magnitude estimation method, with the reference stimulus being a modulus. The hardness value of the modulus was set to 10. Participants operated the haptic display and compared the hardness of a test and reference stimulus. They were able to switch the presented stimuli using a keyboard and experience each stimulus as many times as desired to enable us to acquire answers with little fluctuation from each participant. The participants wore headphones playing pink noise to block out auditory cues. Thirteen test stimuli were presented in random order in a single set of each stimulus group. A total of three sets were performed for each participant with a several minute break between sets. The experiments using stimulus groups α–γ were performed in random order. In total, 117 stimuli (13 stimuli $\times$ 3 sets $\times$ 3 stimulus groups) were tested by each participant. We sacrificed the number of trials in order

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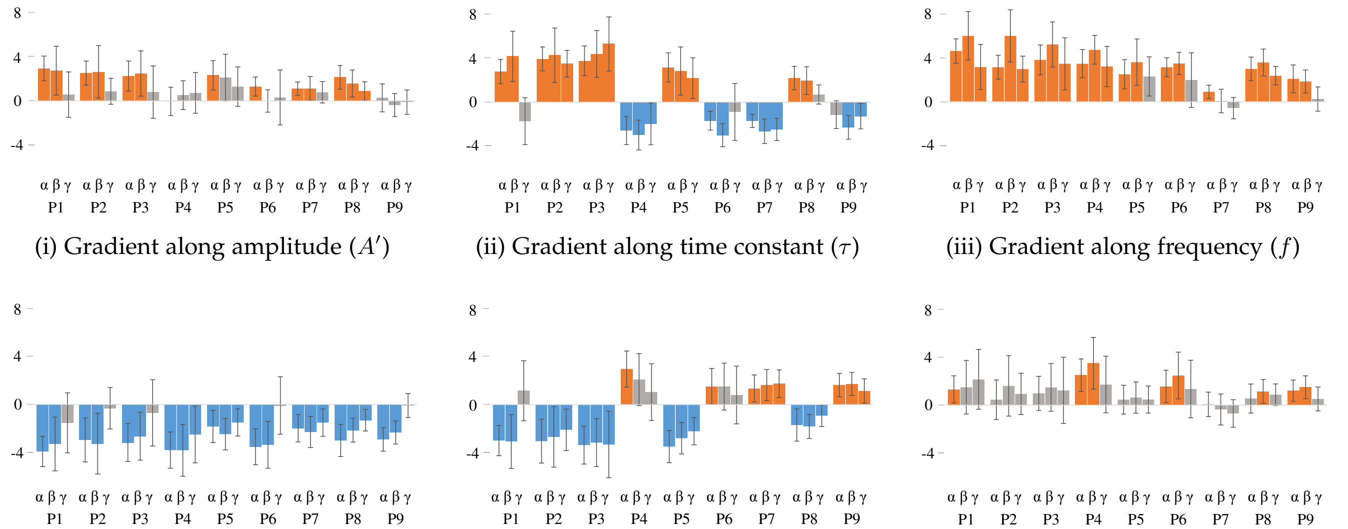

(i) Gradient along amplitude (A') (ii) Gradient along time constant (τ) (iii) Gradient along frequency (f)

(iv) Gradient along mass (m) (v) Gradient along viscosity (c) (vi) Gradient along stiffness (k)

Fig. 5. Gradients of H along m, c, k, A', τ, and f in Experiment 1 for individual participants. α, β, and γ indicate the experimental conditions or stimuli sets. Digits indicate participants 1–9. Error bars indicate 90% confidence intervals. Colored bars indicate a statistically significant gradient ($p < 0.10$).

to remain participants motivated considering the required time for experiment which was 2.5–3 hours. The small number of trials limited the interpretation of the results with the standard significance level of 0.05 or smaller when the results were analyzed for individual participants.

2.5 Participants

Participants were eight right-handed males and one right-handed female in their twenties. All signed a written consent form and were unaware of the objectives of the experiments beforehand.

2.6 Analysis

The answered values for perceived hardness for each participant were fitted to the approximate function of perceived hardness $H(m, c, k)$ and $H(A', \tau, f)$ as follows:

$$H(m, c, k) = p_0 + p_1 m + p_2 c + p_3 k \quad (7)$$
$$H(A', \tau, f) = p_4 + p_5 A' + p_6 \tau + p_7 f \quad (8)$$

where H and p_i were the values of the perceived hardness and regression coefficients, respectively. These linear equations were intended to express the subjective hardness in a small parameter space. We did not simultaneously use the six types of variables as explanatory variables because the mechanical parameters (m, c, and k) and vibration properties (A', τ, and f) are not independent as indicated by (2), (3), and (4).

For these regression analyses, the values of the perceived hardness were geometrically averaged for the individuals, and the values for each physical parameter were normalized to z-scores. Because the parameters could be mutually converted between the physical characteristics of the object and the vibration properties using (2), (3), and (4), the results could be analyzed in both dimensions.

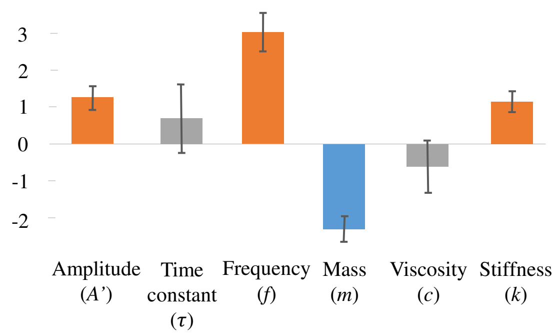

Fig. 4. Mean gradients of H along each parameter in Experiment 1 (virtual stimuli). Error bars indicate the standard errors among the participants. Orange and blue bars are significantly greater and smaller than zero, respectively ($p < 0.001$).

We discussed the contribution of each parameter to hardness perception from the gradients of H along each parameter in the vicinity of the parameters of a reference stimulus. These gradients along the parameters correspond to the regression coefficients of the parameters.

2.7 Results

To check how well the H functions represented the observed data, we calculated the correlation coefficients between the subjective hardness values answered by participants and those estimated by H functions. When $H(m, c, k)$ was used, for each group of α–γ, the mean and standard deviation of correlation coefficients among participants were $r = 0.78 \pm 0.10$, 0.69 ± 0.08, and 0.44 ± 0.16. Moreover, when $H(A', \tau, f)$ was used, the r values were 0.83 ± 0.09, 0.78 ± 0.11, and 0.64 ± 0.18. Comparing the r values of groups α and β (vibration only) and γ (vibration with a virtual wall as the quasi-static reaction force), the existence of the reaction force led to the decrease in the effects of

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the parameters. This is reasonable because participants used both the reaction force from the wall and vibratory cues to judge the hardness for the stimuli in group γ. The contributions of vibratory cues were smaller for group γ than for other groups.

As mentioned in Section 1, we expected that the effects of virtual stimuli would depend on individual participants. Nonetheless, we first show the mean effects of vibration and physical parameters across all participants. Fig. 4 shows the means and standard errors of the gradient of H along each parameter. Except for the viscosity and time constant, the gradients of parameters were significantly greater or smaller than zero ($p < 0.001$). The amplitude and frequency of the vibration and stiffness of virtual objects exhibited positive effects on hardness perception, whereas the mass negatively influenced the same. In contrast, the time constant and viscosity seem to have no effects on hardness perception.

Fig. 5 shows the gradients of H along each vibration property for each individual participant. The error bars indicate 90% confidence intervals. Orange and blue bars indicate statistically significant positive and negative effects, respectively, on hardness perception ($p < 0.10$). Gray bars are insignificant gradients judged by statistical tests of the regression coefficient ($p > 0.10$). Although some of the gradients are significant and others are not, the gradients along the vibration properties and mechanical parameters tended to have the same sign among experiments for groups α–γ. The six types of parameters can be classified into two types: those that exhibited individual differences in the sign of the coefficient, and those that did not. The mass, stiffness, amplitude, and frequency matched the sign of the gradient among all participants. The stiffness, amplitude, and frequency exhibited positive gradients, while the mass had a negative gradient. On the other hand, the viscosity and time constant exhibited individual differences in the sign of the gradient. For the participants in one group, their perceived hardness was positively affected by the viscosity, whereas the participants of the other group found the perceived hardness to be negatively affected by the viscosity. The time constant had the opposite effect on the perceived hardness.

3 Experiment 2: Subjective hardness of real objects

3.1 Spring-damper specimens

We assembled spring-damper specimens [8] to present the damped vibration of a single-frequency component. As shown in Fig. 6, the spring-damper specimen was composed of two aluminum plates, a linear spring (SWY-31-40, SWU-31-40, and SWR-31-40, MISUMI, Japan), and a shock absorber (EMACN1212A, B, and C, MISUMI, Japan). Each aluminum plate was 80 × 80 mm in size, and the upper and lower plates were 3 mm and 5 mm thick, respectively. Three types of springs and shock absorbers with different strengths could be used. Thus, there were five types of specimens, as listed in Table 3. The springs were sufficiently rigid to not be deflected when pushed with a fingertip. Furthermore, the specimens were placed upon a buffer material to muffle their static reaction forces. This made it even more difficult for participants to judge the hardness based on the static deflection of the specimen. From observation

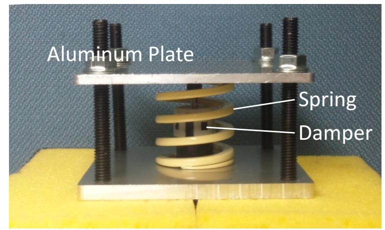

Fig. 6. Spring-damper system consisting of mechanical components. The subject tapped the upper surface of the aluminum plate that was supported by a spring and damper.

TABLE 3
Measured parameters of stimuli used in Experiment 2

Sample	Frequency [Hz]	Time Constant [ms]	Stiffness [N/m]	Viscosity [Ns/m]
I	381	120	15700	0.047
II	384	200	15700	0.026
III	381	69	15700	0.078
IV	354	130	7830	0.024
V	440	140	24700	0.047

using tapping tests and an accelerometer on the aluminum surface, the vibration caused by tapping included a single major frequency that varied with the combination of a spring and damper.

3.2 Tasks

Participants tapped the specimens and experienced their damped vibrations. As in Experiment 1, they evaluated the subjective hardness of a test stimulus as a positive real number based on the magnitude estimation method. The hardness value of the reference stimulus, which was sample I, was defined as 10. Participants were able to experience each stimulus as many times as desired. The stimuli were presented in random order. Five stimuli were tested in a single set and a total of three sets were performed for each participant.

3.3 Participants

The same participants were used as in Experiment 1. They wore headphones playing pink noise to block out audio cues. The order in which Experiments 1 and 2 were performed was balanced among the participants.

3.4 Analysis

The answered values for perceived hardness were fitted to the approximate function for perceived hardness $H(c, k)$ and $H(\tau, f)$ as follows:

$$H(c,k) = q_0 + q_1 c + q_2 k \quad (9)$$

$$H(\tau,f) = q_3 + q_4 \tau + q_5 f \quad (10)$$

where q_i was the regression coefficient. The values of the perceived hardness were geometrically averaged for individuals, and the physical parameters of the vibration stimuli were normalized to z-scores. Similar to Experiment 1, we discussed the contribution of each parameter from the gradients of H along each parameter in the vicinity of the parameters of a reference stimulus.

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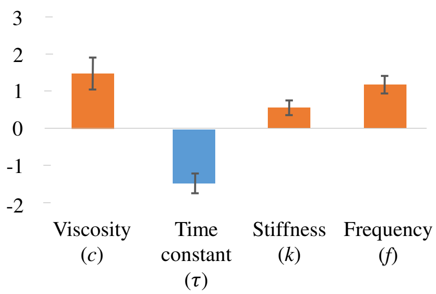

Fig. 7. Gradients of H along c, k, τ, and f in Experiment 2 (actual stimuli). Error bars indicate the standard errors among the participants.

3.5 Results

Similar to Experiment 1, to check how well H functions fit with the observations, we calculated the correlation coefficients between the answered and estimated subjective hardness values for individual participants. For $H(c, k)$ and $H(\tau, f)$, the means and standard deviations of the correlation coefficients among participants were $r = 0.72 \pm 0.21$ and 0.70 ± 0.29. Both H functions equally estimated the answered hardness values.

Fig. 7 shows the gradients of H along each parameter. Because no individual differences were observed, the results for all participants are averaged and shown in a single figure. Error bars indicate the standard errors among the participants. The stiffness, frequency, and viscosity exhibited positive gradients and the time constant exhibited a negative gradient. The effects of the stiffness and frequency matched those in the virtual environment. There were no individual differences in the sign of the coefficient on the viscosity and time constant, unlike the case of the virtual object.

4 Discussion

Here, we discuss the effect of each parameter on the perceived hardness. The consistency or inconsistency of the effect of each parameter among participants in virtual and real environments is shown in Table 4. When the signs of all significant gradients along a specific parameter in Fig. 5 are the same, we judged the effect of that parameter as consistent. Note that individual differences in the effect of some parameters were observed only in the experiment using virtual stimuli.

4.1 Consistent effect of frequency

Vibration stimuli with higher frequency induced greater perceived hardness. Regardless of whether the object was real or virtual, the effect of the frequency was consistent among participants. This result for the frequency agrees with previous studies [1], [2]. The vibration frequency is an effective parameter for controlling the perceived hardness of real and virtual objects.

4.2 Consistent effect of stiffness

Greater stiffness enhanced the perceived hardness for both the virtual and real objects. In our experiment, the stiffness of the objects contributed only to the characteristics of damped natural vibration, and not to the static reaction force. Even in the experiment using real objects, the participants were unlikely to use the static reaction forces to judge hardness because they were muffled by the soft buffer material. Therefore, humans can discriminate between differences in hardness without the static reaction force, by using the vibration stimulus caused by tapping. This result supports the method of presenting the stiffness change of virtual objects based on vibratory feedbacks with ungrounded devices that cannot present a static reaction force.

Nonetheless, according to Fig. 5(vi), for virtual stimuli, only a few participants felt that the stiffness k exhibited positive effects on the perceived hardness. For other participants, the effects of k were insignificant. This phenomenon can be explained as follows. As in (4), the increase in k increases the vibratory frequency f, which leads to a greater perceived hardness. Simultaneously, as in (2), the increase in k decreases the vibration amplitude A'. According to Fig. 5(i), a decrease in A' leads to a smaller perceived hardness. Hence, the effects of f and A' mitigate the effects of each other on perceived hardness, and as a result, the effect of k was moderate.

4.3 Inconsistent effects of vibration amplitude between virtual and real objects

In this experiment, the vibration amplitude was the product of the contact speed v and the amplitude per contact speed A'. Here, we discuss the effect of A'.

In the virtual object, the perceived hardness increased as the amplitude of the vibration stimulus increased. The results show that the amplitude is an effective parameter for controlling the hardness of the virtual object. In the case of the real object, the higher stiffness decreases the amplitude of the damped natural vibration of the object surface. Here, we find that the amplitude has an opposite effect on the real and virtual cases.

The above mentioned inconsistency was potentially caused by the gap between the vibration model of (1) and the fingertip's vibration caused by tapping real objects. In the vibration model, the fingertip and target object collectively vibrate as one object; however, the displacements of their surfaces may not match in real situations [13], unlike the model used in the present study. For example, when tapping harder objects with a tool, the vibration amplitude on the tool surface increases [6] in a manner opposite to (1). The selection of an appropriate model to cause a perceptually sound effect is still an open question.

4.4 Inconsistent effects of time constant between virtual and real objects

The effect of the time constant exhibited individual differences in the virtual environment. When the time constant increased, five of the nine participants reported an increase in the perceived hardness, and the others reported its decrease. Therefore, the time constant is not an effective parameter for stable control of the perceived hardness of virtual objects. However, since the effect of the time constant is significant for each individual, the time constant can express the difference in the material of virtual objects.

TABLE 4
Perceptual effects of six types of parameters on subjective hardness. "Positive" or "negative" indicate that the parameter is positively or negatively correlated with the subjective hardness. "Inconsistent" indicates that the effect of the parameter depends on individuals.

	Frequency (f)	Time constant (τ)	Stiffness (k)	Viscosity (c)	Amplitude (A')	Mass (m)
Virtual stimulus	Positive	Inconsistent among participants	Positive	Inconsistent among participants	Positive	Negative
Real specimen	Positive	Negative	Positive	Positive	-	-

The reason for the individual differences is unclear. Our experimental design was not to investigate the reason for the individual differences in the virtual environment. We had considered that such individual differences were attributed to a virtual wall, however, we did not find any substantial differences between conditions with and without the virtual wall. The same perceptual phenomena were observed in our past study [11] where a commercial haptic interface and a rendering algorithm with the effect of a virtual wall were used. Hence, the observation of these differences was not unexpected in the present study.

On the other hand, in the case of the real damped natural vibration, the effect of the time constant did not exhibit individual differences. A longer time constant decreased the perceived hardness for all participants.

4.5 Effect of viscosity via effect on the time constant

The increase in the viscosity of real objects had a consistent effect of increasing the perceived hardness for all participants. On the other hand, the effect of the viscosity exhibited individual differences in the experiment using virtual objects. The inconsistency of the effect of the viscosity change in virtual objects was observed in our previous studies using a commercial force display [8], [11]. As shown in Fig. 5(ii) and (v), all participants reported that the viscosity and time constant influenced their perceived hardness in the opposite manner. These results were consistent with the inversely proportional relationship between the viscosity and time constant in (3). Thus, it is assumed that the effect of the viscosity is caused by the resultant time constant change.

4.6 Effect of mass of the virtual object

The virtual object with the smaller mass was felt as the harder object. As shown in (2), (3), and (4), the decrease in the mass increases the amplitude and frequency of the vibration stimuli, and decreases its time constant. The increase in the frequency and amplitude enhances the perceived hardness, as mentioned in previous sections. On the other hand, the decrease in the time constant can result in either the increase or decrease in the perceived hardness depending on the participant. In summary, an increase in the mass induces greater perceived hardness with the composite effect of three vibration parameters.

4.7 Reaction force of the spring wall decreases the apparent roles of vibration properties

The results of experiments β (without spring-wall) and γ (with spring-wall) showed that the sign of the gradient along each vibration property was not changed by adding the quasi-static force. However, the quasi-static force increased the perceived hardness, and relatively decreased the effects of the vibration properties. This is because the combination of the vibration and force results in the decrease of each contribution ratio against the whole of the perceived hardness.

5 Conclusion

We researched the effects of parameters related to the damped natural vibration by investigating the perceived hardness based on tapping real and virtual objects.

For virtual objects, vibration stimuli were produced through a momentum-conservation model where the virtual fingertip and object collectively vibrated after the collision. The decrease in the virtual mass and increase in the virtual stiffness, vibration amplitude, and frequency induced greater perceived hardness perception. The viscosity and time constant of the virtual stimuli exhibited inconsistent effects among participants and influenced perceived hardness in the opposite manner for each participant.

We validated the effects of the viscosity, stiffness, time constant, and frequency by using the real objects. The results showed that the effects of the stiffness and frequency were consistent between the real and virtual objects. On the other hand, the effects of the viscosity and time constant were different from those of the virtual objects. When real objects were used, an increase in the viscosity and decrease in the time constant led to greater hardness perception.

The stiffness and frequency are effective indexes for describing the perceived hardness of real/virtual environment and individuals. The inconsistency of the results between the virtual and real objects suggests that there remains a gap in the dynamic phenomena between the vibration of the real object case and the reproduced stimulus on the haptic interface. Developers of haptic applications should remember that the general physical model that induces perceptually consistent effects with real situations remains unsolved.

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