A novel adaptive beamformers for MEG source reconstruction effective when large background brain activities exist

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Interferences affecting to MEG sensor data

Low-rank interference

High-rank interference

Electronics

Heartbeat

Eyeblinks

Dental & Jaw Muscles

Spontaneous brain activity

no specific spatial, temporal, and frequency patterns
Data model

\[ b(t) = b_s(t) + b_I(t) + n(t) \]

Measured magnetic field \[ b(t) \]
Signal magnetic field \[ b_s(t) \]
Sensor noise \[ n(t) \]
Background interference \[ b_I(t) \]

Definitions

Data covariance: \( R = \langle b(t)b^T(t) \rangle \)
Signal covariance: \( R_S = \langle b_s(t)b_s^T(t) \rangle \)
Interference plus noise covariance: \( R_{i+n} = \langle (b_I(t) + n(t))(b_I(t) + n(t))^T \rangle \)
Data model

Task: \( b(t) = b_s(t) + b_I(t) + n(t) \)

Control: \( b_C(t) = b_I(t) + n(t) \)

Covariance matrix relations

Task: \( R = R_S + R_{i+n} \)

Control: \( R_C = R_{i+n} \)

Problem  How to obtain source reconstruction free from the influence of \( b_I(t) \)

We propose:

(1) Prewhitening beamforming
(2) Partitioned factor analysis + adaptive beamforming
Prewhitening estimation of signal covariance

Calculate \( \tilde{R} = R_C^{-1/2} RR_R^{1/2} \)

(Tilde is used to indicate the prewhitened version of a matrix)

\[
\tilde{R}_S = \sum_{j=1}^{Q} \gamma_j u_j u_j^T \implies \tilde{R} = \sum_{j=1}^{Q} (\gamma_j + 1) u_j u_j^T + \sum_{j=Q+1}^{M} u_j u_j^T.
\]

\( \uparrow \)

\( \tilde{R} \) has signal-level eigenvalues greater than 1, and their eigenvectors are equal to those of \( \tilde{R}_S \)

Signal covariance estimation

\[
\hat{R}_S = R_C^{1/2} \left[ U_S U_S^T (\tilde{R} - I) \right] R_C^{1/2} = R_C^{1/2} \left[ \sum_{j=1}^{Q} (\gamma_j - 1) u_j \right] R_C^{1/2}
\]

\( \uparrow \)

\( U_S = [u_1, \ldots, u_Q] \)

For details, poster P-163, (session P4-1) 8/22 3:00—5:00PM
Prewhitening beamforming

Signal covariance estimation

\[ \hat{R}_S = R_C^{1/2} \left[ \sum_{j=1}^{Q} (\gamma_j - 1)u_j \right] R_C^{1/2} \]

Signal time course estimation

\[ \hat{b}_S(t) = R_C^{1/2} \left[ \sum_{j=1}^{Q} (\gamma_j - 1)u_j \right] R_C^{-1/2} b_C(t) \]

Prewhitening beamforming

\[ \hat{s}_{PW}(r, t) = \frac{l^T(r)(\hat{R}_S + \mu I)^{-1} \hat{b}_S(t)}{l^T(r)(\hat{R}_S + \mu I)^{-1} l(r)} \]
Partitioned factor analysis (PFA)  
Variational Bayesian Factor Analysis (VBFA)

Data model:

measured data $\rightarrow y_n = Ax_n + v_n$

mixing matrix factors sensor noise

Prior probability:

$p(x_n) = N(x_n \mid 0, I)$  
$p(A) = \prod_{m,l} N(A_{m,l} \mid 0, \lambda_m \alpha_l)$  
$p(v_n) = N(v_n \mid 0, \Lambda)$

Derivation of posterior probability:

$p(A \mid y) = \arg \max_{q(A \mid y)} F(q)$  
$p(x \mid y) = \arg \max_{q(x \mid y)} F(q)$

Here $F(q) = \int dx dA [\log p(x, y, A) - \log q(x \mid y) - \log q(A \mid y)]$
Variational Bayes EM algorithm

E step: \( p(x \mid y) = \arg \max_{q(x|y)} F(q) \)

\[
\Gamma = \tilde{A}^T \Lambda \tilde{A} + M \Psi^{-1} + I
\]

\( \tilde{x}_n = \Gamma^{-1} \tilde{A}^T \Lambda y_n \)

for \( p(x_n \mid y_n) = N(x_n \mid \tilde{x}_n, \Gamma) \)

M step: \( p(A \mid y) = \arg \max_{q(A|y)} F(q) \)

\[
\tilde{A} = R_{yx} (R_{xx} + \alpha)^{-1}
\]

\( \Psi = R_{xx} + \alpha \)

for \( p(A \mid y_n) = \prod_{m,l} \frac{1}{M} N(a_m \mid \tilde{a}_m, \lambda_m \Psi) \)

\[
\alpha = \arg \min_{\alpha} F(q) \quad \Rightarrow \quad \alpha = \frac{1}{M} \text{diag}(\tilde{A}^T \Lambda \tilde{A} + \Psi^{-1})
\]

\( \Lambda = \arg \min_{\Lambda} F(q) \quad \Rightarrow \quad \Lambda = \frac{1}{N} \text{diag}(R_{yy} - \tilde{A} \tilde{R}_{yx}^T) \)
Partitioned factor analysis (PFA)

Stimulus-Evoked Factor Analysis (SEFA)

Data model

Control: $y_n = Bu_n + v_n$

Task: $y_n = Ax_n + Bu_n + v_n$

(1) Apply VBFA to control data and obtain $\bar{B}$, and the noise precision $\Lambda$.

(2) Apply VBFA to task data, and obtain

$$\hat{b}_S(t) = E_A E_x[Ax_n] = \bar{A} \bar{x}_n$$

$$\hat{R}_S = E_A E_x[(Ax_n)(Ax_n)^T] = \bar{A} R_{xx} \bar{A}^T + \Lambda^{-1} \text{tr}[R_{xx}(R_{xx} + \alpha)^{-1}]$$

(3) Use these estimated results for adaptive beamforming.

$$\hat{s}_{PFA}(r, t) = \frac{l^T(r) \hat{R}_S^{-1} \hat{b}_S(t)}{l^T(r) \hat{R}_S^{-1} l(r)}$$

For details, visit poster P-116, (session P3-1) 8/21 10:00—12:00PM
Generate 100 background sources with random time courses
Data without background sources

Data with background sources

⇒

⇒

⇒

PFA

prewhitening

conventional

conventional
Computer Simulation: Robustness to insufficient control data

Tc: 400 time points

PFA

Tc: 200

PW

Tc: 100
Auditory evoked field

Conventional PW

PFA

Conventional PW

PFA
Non-ideal scenarios for two-condition measurements

Some target sources are also active in the control condition

Task: \( b(t) = b_S(t) + b_I(t) + n(t) \)

Control: \( b_C(t) = b_S(t) + b_I(t) + n(t) \)

Some sources are active only in the control condition

Task: \( b(t) = b_S(t) + b_I(t) + n(t) \)

Control: \( b_C(t) = b_I(t) + n(t) + b_\Delta(t) \)

These additional terms may affect the performance of PFA

Prewhitening technique is robust in these scenarios, as discussed at poster P-163, (session P4-1) 8/22 3:00—5:00PM
Computer simulation for non-ideal scenarios

Relative source intensity

<table>
<thead>
<tr>
<th>Source</th>
<th>control</th>
<th>task</th>
</tr>
</thead>
<tbody>
<tr>
<td>First source</td>
<td>1</td>
<td>1.5</td>
</tr>
<tr>
<td>Second source</td>
<td>1.5</td>
<td>1.5</td>
</tr>
<tr>
<td>Third source</td>
<td>1</td>
<td>1.5</td>
</tr>
</tbody>
</table>

96 paired data sets generated
Signal-to-Interference Ratio: 0.5

Task

Control

Subtracted

True source location

Signal-to-Interference Ratio: 0.25

Task

Control

Subtracted

Source location
Signal-to-Interference Ratio: 0.33

Relative source intensity

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The diagram shows the relative source intensity for different tasks, with the signal-to-interference ratio calculated as 0.33. The table compares the intensity for control and task conditions for each source.
Summary

• We propose two methods to obtain source reconstruction robust to the existence of large background brain activity.
  1) Prewhitening beamforming
  2) Partitioned factor analysis + adaptive beamforming

• Both methods are effective in ideal two-condition measurements.

• Partitioned factor analysis is very robust to a case where the number of time samples is small.

• Both methods are significantly robust to non-ideal scenarios of two-condition experiments. However, prewhitening method gives smaller source estimation bias than the PFA method.