Recent advances in the analysis of biomagnetic signals

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Application of spatial filter techniques to biomagnetic source localization
Definitions

- data vector: \( b(t) = \begin{bmatrix} b_1(t) \\ b_2(t) \\ \vdots \\ b_M(t) \end{bmatrix} \)
  
- \( b_j(t) \): the \( j \)th sensor recording at \( t \)

- data covariance matrix: \( D = \langle b(t)b^T(t) \rangle \)

\( \langle \cdot \rangle \) represents time average
Source moment

- magnitude at \( \mathbf{r} = [x, y, z] \)
  
  and at \( t \): \( s(\mathbf{r}, t) \)

- orientation:

  \[ \eta(\mathbf{r}, t) = [\eta_x(\mathbf{r}, t), \eta_y(\mathbf{r}, t), \eta_z(\mathbf{r}, t)] \]

- source moment vector:

  \[
  \mathbf{s}(\mathbf{r}, t) = \mathbf{s}(\mathbf{r}, t) \begin{bmatrix}
  \eta_x(\mathbf{r}, t) \\
  \eta_y(\mathbf{r}, t) \\
  \eta_z(\mathbf{r}, t)
  \end{bmatrix} = \begin{bmatrix}
  s_x(\mathbf{r}, t) \\
  s_y(\mathbf{r}, t) \\
  s_z(\mathbf{r}, t)
  \end{bmatrix}
  \]

  \( \eta_x = \sin \theta \cos \varphi \)

  \( \eta_y = \sin \theta \sin \varphi \)

  \( \eta_z = \cos \theta \)
Lead field vector for the $j$ th sensor

\[ l_j(r) = [l_j^x(r), l_j^y(r), l_j^z(r)] \]

Lead field matrix for the whole sensor array

\[ L(r) = \begin{bmatrix} l_1(r) \\ l_2(r) \\ \vdots \\ l_M(r) \end{bmatrix} = \begin{bmatrix} l_1^x(r) & l_1^y(r) & l_1^z(r) \\ l_2^x(r) & l_2^y(r) & l_2^z(r) \\ \vdots & \vdots & \vdots \\ l_M^x(r) & l_M^y(r) & l_M^z(r) \end{bmatrix} \]
Basic relationship

\[ b_j(t) = \int l_j(r)s(r,t)\,dr \]

or

\[ b(t) = \int L(r)s(r,t)\,dr \]

Problem of source localization:

**Estimate** \(s(r,t)\) **from the measurement** \(b(t)\)
Spatial filter

\[
\hat{s}(r, t) = w^T(r)b(t) = [w_1(r), \ldots, w_M(r)] \begin{bmatrix} b_1(t) \\ \vdots \\ b_M(t) \end{bmatrix}
\]

\[\uparrow \quad \uparrow \]

estimate of \( s(r, t) \) \quad weight vector

(neglecting the explicit time notation)

\[
b = \int L(r)s(r)dr \\
\hat{s}(r) = w^T(r)b \quad \rightarrow \quad \hat{s}(r) = \int \underbrace{w^T(r)L(r')}_\mathcal{R}(r, r') s(r')dr'
\]

Resolution kernel
Resolution kernel:  \[ \hat{s}(r) = \int \mathbb{R}(r, r')s(r')dr' \]

The weight \( w(r) \) must be chosen so that the resolution kernel has a reasonable shape.

What is reasonable shape?

- peak at \( r \)
- small main-lobe width
- low side-lobe amplitude
Non-adaptive weight

\( w(r) \) is data independent

Adaptive weight

\( w(r) \) is data dependent
Data-independent (non-adaptive) weight

• Spatial resolution is limited by sensor-array configuration.

• Final results are not influenced by source temporal correlation.

Data-dependent (adaptive) weight

• Spatial resolution can exceed the limit imposed by the sensor-array configuration.

• Strong temporal correlation among source activities severely degrade the quality of final estimation results.
Data-independent (non-adaptive) weight

minimum-norm estimate (Hamalainen et al.)

The weight $w(\mathbf{r})$ is obtained by

$$
\min \int \left[ \mathbb{R}(\mathbf{r}, \mathbf{r}') - \delta(\mathbf{r} - \mathbf{r}') \right]^2 d\mathbf{r}'
\downarrow
$$

$$
\mathbf{w}^T(\mathbf{r}) = \mathbf{L}^T(\mathbf{r}) \mathbf{G}^{-1}, \quad \text{where } G_{i,j} = \int l_i(\mathbf{r}) l_j(\mathbf{r}) d\mathbf{r}
$$

Inverse solution: $\hat{s}(\mathbf{r}) = \mathbf{L}^T(\mathbf{r}) \mathbf{G}^{-1} \mathbf{b}$
Property of $G$ matrix

$$G_{i,j} = \int l_i(r) l_j(r) dr$$

Overlaps of sensor lead fields is large
$G$ is poorly conditioned

Overlaps of sensor lead fields is small
$G$ has a small condition number
$G$ is usually calculated by introducing pixel grid $r_j$

$$b = \int L(r)s(r) \, dr = \sum_{j=1}^{N} L(r_j) s(r_j)$$

$$= \begin{bmatrix} L(r_1), & \cdots, & L(r_N) \end{bmatrix} \begin{bmatrix} s(r_1) \\ \vdots \\ s(r_N) \end{bmatrix} = L_N s_N$$

Therefore $G = L_N L_N^T$ and

$$w^T(r) = L^T(r) (L_N L_N^T)^{-1}$$

or

$$w^T(r) = L^T(r) (L_N L_N^T + \gamma I)^{-1} \quad \text{[regularized version]}$$
Minimum-norm weight with normalized lead field

\[ w(r) = (\bar{L}_N^T \bar{L}_N + \gamma I)^{-1} L(r) / \|L(r)\| \]

where \( \bar{L}_N = \left[ \frac{L(r_1)}{\|L(r_1)\|}, \ldots, \frac{L(r_N)}{\|L(r_N)\|} \right] \)

Minimum-norm estimate with normalized weight

Calculate \( \hat{q}(r) = \|w^T(r)b\|^2 / \|w(r)\|^2 \)

Dale et al.,
Valdes et al.
no normalization

normalized lead field used

normalized weight used
Source imaging experiments

- Auditory-evoked field were measured using a 148-channel whole-head sensor array (Magnes 2500).

**Stimulus:** 1-kHz pure tone applied to subject’s left ear

**Data acquisition:** 1 kHz sampling frequency, 1-400 Hz bandpass filtering, 100 epochs averaged
Linear-estimation-based methods

• LORETTA: impose the maximum-smoothness constraint. (Pascual-Marqui et al.)

• fMRI constraint: constrain solution based on fMRI results. (Dale et al.)

• FOCUSS: obtain a focal solution iteratively. (Gorodnitsky et al.)

• Bayesian approach: impose prior assumptions. (Schmidt et al.)

• $l_1$-norm approach: use the $l_1$-norm, instead of using the $l_2$-norm. (Matuura et al., Uutela et al., Beucker et al.)
Data dependent (adaptive) weight

minimum-variance beamformer

\[ \min_{w} w^{T} D w \text{ subject to } w^{T} l(r, \eta) = 1 \]

\[ w^{T}(r, \eta) = \frac{l^{T}(r, \eta) D^{-1}}{l^{T}(r, \eta) D^{-1} l(r, \eta)} \quad \text{and} \quad \hat{s}(r, \eta) = \frac{l^{T}(r, \eta) D^{-1} b}{l^{T}(r, \eta) D^{-1} l(r, \eta)} \]

\[ \|L(r)\eta \leftarrow \text{beamformer pointing orientation} \]
Generalized Wiener estimate:

\[
\hat{s}_N = RL_N^T (L_N R L_N^T + C)^{-1} b
\]

\[\uparrow\]
source covariance matrix
\[\uparrow\]
noise covariance matrix

Use \( R \approx I \), and \( C \approx \gamma I \) (assume no prior-information)

\[\downarrow\]

\[
\hat{s}_N = L_N^T (L_N L_N^T + \gamma I)^{-1} b
\]

minimum-norm estimate
Generalized Wiener estimate:

\[ \hat{s}_N = RL_N^T(L_N RL_N^T + C)^{-1}b \]

- use \( D = L_N RL_N^T + C \), and
- assume off-diagonals of \( R \) are all zero

\[ \downarrow \]

\[ \hat{s}_p = R_{pp}L_P^TD^{-1}b \]

Relashonship, \( \hat{s}_p^2 = R_{pp} \) leads to \( R_{pp} = 1/(L_P^TD^{-1}l_P) \)

\[ \hat{s}_p = (L_P^TD^{-1}b)/(L_P^TD^{-1}l_P) \]

minimum-variance estimate
Problems when applying MV beamformer to MEG source localization

(1) How to determine beamformer orientation $\eta$.

(2) $\|L(r)\|$ – norm artifact.

(3) Output SNR degradation due to the matrix inversion.
How to determine beamformer orientation $\eta$?

The weight $w(r, \eta)$ is calculated for $r$ and $\eta$.

- search all directions (Robinson and Vrba).
- use the MUSIC algorithm to determine $\eta$ (Sekihara et al.).
- use vector beamformer formulation. (Spencer et al. Van Veen et al.)

What happens if the beamformer orientation is different from the source orientation?

$\Downarrow$

Severe signal-intensity loss
Computer simulation for calculating beamformer angular response

A single source exists 6-cm below the sensor array

beamformer orientation different from the source orientation with $\delta \theta$

37-channel sensor array

normalized value

Calculated angular response
A single source exist at $r$ with orientation $\eta$ and time course $s(t)$

measured data: $b(t) = [\eta_x l_x(r) + \eta_y l_y(r) + \eta_z l_z(r)]s(t)$

When beamformer weight has a wrong orientation $\eta'(\neq \eta)$

$\mathbf{w}^T(r, \eta')b(t) = \mathbf{w}^T(r, \eta')[\eta_x l_x(r)s(t) + \eta_y l_y(r)s(t) + \eta_z l_z(r)s(t)]$

$\uparrow$

virtual three coherent sources

$\approx 0$
Vector beamformer formulation

Three weight vectors $[w_x(r), w_y(r), w_z(r)]$ detects the $x$, $y$, and $z$ component of the source moment, separately.

$$\begin{align*}
\min w_x^T D w_x \text{ subject to } w_x^T l(r, f_x) &= 1, \quad w_x^T l(r, f_y) = 0, \quad w_x^T l(r, f_z) = 0 \\
\min w_y^T D w_y \text{ subject to } w_y^T l(r, f_x) &= 0, \quad w_y^T l(r, f_y) = 1, \quad w_y^T l(r, f_z) = 0 \\
\min w_z^T D w_z \text{ subject to } w_z^T l(r, f_x) &= 0, \quad w_z^T l(r, f_y) = 0, \quad w_z^T l(r, f_z) = 1
\end{align*}$$

$$\downarrow$$

$$[w_x(r), w_y(r), w_z(r)] = D^{-1} L(r) [L^T(r) D^{-1} L(r)]^{-1}$$

and

$$\hat{s}(t) = [L^T(r) D^{-1} L(r)]^{-1} L^T(r) D^{-1} b(t)$$

$$f_x = [1, 0, 0]^T, \quad f_y = [0, 1, 0]^T, \quad f_z = [0, 0, 1]^T$$
Calculated beamformer angular response

- **Cosine curve**
- **Scalar beamformer**
- **Vector beamformer**
Severe artifacts arise near the center of the sphere, when the spherical conductor model is used.

To avoid these artifacts

• use normalized lead field (Van Veen et al.)
• use normalized weight (Robinson et al., Sekihara et al.)
assumed source waveform

relative magnitude

generated magnetic field

37-channel sensor array

reconstruction region

sphere for the forward modeling
Time-averaged reconstruction \( \left\langle \| \hat{s}(r, t) \|^2 \right\rangle \)

- no normalization
- normalized lead field used
Borgiotti-Kaplan beamformer

$$\min_w w^T D w \text{ subject to } w^T w = 1$$

$$\downarrow$$

$$w^T (r, \eta) = \frac{l^T (r, \eta) D^{-1}}{\sqrt{l^T (r, \eta) D^{-2} l(r, \eta)}}$$

and

$$\langle \| \hat{s}(r, \eta) \|^2 \rangle = \frac{l^T (r, \eta) D^{-1} l(r, \eta)}{l^T (r, \eta) D^{-2} l(r, \eta)}$$
Vector extension of Borgiotti-Kaplan beamformer

\[
\begin{align*}
\min w_x^T D w_x \text{ subject to } & w_x^T w_x = 1, \ w_x^T l(r, f_y) = 0, \ w_x^T l(r, f_z) = 0 \\
\min w_y^T D w_y \text{ subject to } & w_y^T l(r, f_x) = 0, \ w_x^T w_y = 1, \ w_y^T l(r, f_z) = 0 \\
\min w_z^T D w_z \text{ subject to } & w_z^T l(r, f_x) = 0, \ w_z^T l(r, f_y) = 0, \ w_z^T w_z = 1 \\
& \downarrow \\
& \text{let } \mu=x, \ y \text{ or } z \\
& w_\mu(r) = \frac{D^{-1} L(r) [L^T(r)D^{-1} L(r)]^{-1} f_\mu}{\sqrt{f_\mu^T \Omega f_\mu}} \\
\hline
& \Omega = [L^T(r)D^{-1} L(r)]^{-1} L^T(r)D^{-2} L(r) [L^T(r)D^{-1} L(r)]^{-1}
\end{align*}
\]
Time-averaged reconstruction $\left( \| \hat{s}(r, t) \|^2 \right)$

- Normalized lead field used
- Normalized weight used
  (B-K beamformer results)
Resolution kernel for BK beamformer
Output SNR degradation for spatio-temporal reconstruction

Signal-to-noise ratio of the beamformer output is severely degraded even by a small error in the estimated lead field

This is caused by the use of direct matrix inversion

To avoid this,

- use regularized inverse (Robinson et al.)
- use eigenspace projection (Sekihara et al.)
Eigenspace projection

Eigendecomposition of $D$

$$
D = U \begin{bmatrix}
\lambda_1 & 0 & \cdots & 0 \\
0 & \ddots & & 0 \\
\vdots & & \ddots & \vdots \\
0 & \cdots & 0 & \lambda_M
\end{bmatrix} U^T = U \begin{bmatrix}
\Lambda_S & 0 \\
0 & \Lambda_N
\end{bmatrix} U^T
$$

$$
U = \begin{bmatrix}
e_1, \ldots, e_P \\
E_S \\
e_{P+1}, \ldots, e_M \\
E_N
\end{bmatrix} = [E_S \mid E_N]
$$

Extension to eigenspace projection beamformer

$$
\bar{w}_\mu = E_S E_S^T w_\mu, \quad \text{where } \mu = x, y \text{ or } z
$$
Output of eigenspace-projected BK beamformer

$$\propto \frac{[l^T(r) \Gamma_S l(r)]^2}{[l^T(r) \Gamma_S^2 l(r)]}$$
where $\Gamma_S = E_S \Lambda_S^{-1} E_S^T$, $\Gamma_N = E_N \Lambda_N^{-1} E_N^T$

Output of BK beamformer

$$\propto \frac{[l^T(r) \Gamma_S l(r)]^2}{[l^T(r) \Gamma_S^2 l(r) + \varepsilon^T \Gamma_N^2 \varepsilon]}$$

overall error in estimating $l(r)$

Even when $\varepsilon$ is small, $\varepsilon^T \Gamma_N^2 \varepsilon$ may not be small
assumed source waveform

37-channel sensor array

three sources

reconstruction region

sphere for the forward modeling

generated magnetic field
Spatio-temporal reconstruction by vector-extended BK beamformer
Spatio-temporal reconstruction by vector-extended BK beamformer with regularized inverse, $(D + \gamma I)^{-1}$.
Spatio-temporal reconstruction by vector-extended BK beamformer with eigen-space projection.
Eigen-space projection does not preserve the null constraints

That is,

\[
\begin{align*}
\left[ E_s E_s^T w_x \right]^T l_y(r) & \neq 0, & \left[ E_s E_s^T w_x \right]^T l_z(r) & \neq 0, \\
\left[ E_s E_s^T w_y \right]^T l_x(r) & \neq 0, & \left[ E_s E_s^T w_y \right]^T l_z(r) & \neq 0, \\
\left[ E_s E_s^T w_z \right]^T l_x(r) & \neq 0, & \left[ E_s E_s^T w_z \right]^T l_y(r) & \neq 0.
\end{align*}
\]

This fact does not cause a problem.
(Poster: 167b)
right auditory cortex activation

left auditory cortex activation

correlation coefficient: 0.97
Summary

• Adaptive spatial filter techniques can provide a spatial resolution higher than that of non-adaptive techniques.

• This is because the spatial resolution for non-adaptive techniques is limited by the sensor configuration but adaptive techniques can exceed this limit.

• Correlated source activities, however, affect the quality of the results obtained by adaptive techniques.

Therefore

Adaptive techniques may be suited to observe relatively small cortical regions with high spatial resolution, and non-adaptive techniques may be suited to observe whole-brain activities.
Collaborators

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