MEG adaptive beamformer
source reconstruction technique
in the presence of correlated sources

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This talk presents results of the investigation that evaluates the performance of the MEG adaptive beamformer technique in the presence of correlated sources.
Data vector: \( \mathbf{b}(t) = \begin{bmatrix} b_1(t) \\ b_2(t) \\ \vdots \\ b_M(t) \end{bmatrix} \)

- \( b_j(t) \): the \( j \)th sensor recording at \( t \)

Data covariance matrix: \( \mathbf{D} = \langle \mathbf{b}(t)\mathbf{b}^T(t) \rangle \)

\( \langle \cdot \rangle \) represents time average
Source moment

- magnitude at \( \mathbf{r} = [x, y, z] \)
  and at \( t \): \( s(\mathbf{r}, t) \)

- orientation:
  \[ \eta(\mathbf{r}, t) = [\eta_x(\mathbf{r}, t), \eta_y(\mathbf{r}, t), \eta_z(\mathbf{r}, t)] \]

- source moment vector:
  \[
  \mathbf{s}(\mathbf{r}, t) = s(\mathbf{r}, t) \begin{bmatrix} \eta_x(\mathbf{r}, t) \\ \eta_y(\mathbf{r}, t) \\ \eta_z(\mathbf{r}, t) \end{bmatrix} = \begin{bmatrix} s_x(\mathbf{r}, t) \\ s_y(\mathbf{r}, t) \\ s_z(\mathbf{r}, t) \end{bmatrix}
  \]

\[
\eta_x = \sin \theta \cos \phi \\
\eta_y = \sin \theta \sin \phi \\
\eta_z = \cos \theta
\]
Lead field vector for the $j$ th sensor

$$I_j(r) = [l_j^x(r), l_j^y(r), l_j^z(r)]$$

Lead field matrix for the whole sensor array

$$L(r) = \begin{bmatrix} I_1(r) \\ I_2(r) \\ \vdots \\ I_M(r) \end{bmatrix} = \begin{bmatrix} l_1^x(r) & l_1^y(r) & l_1^z(r) \\ l_2^x(r) & l_2^y(r) & l_2^z(r) \\ \vdots & \vdots & \vdots \\ l_M^x(r) & l_M^y(r) & l_M^z(r) \end{bmatrix} = \begin{bmatrix} l_x(r), l_y(r), l_z(r) \end{bmatrix}$$
Basic relationship

\[ b_j(t) = \int l_j(r)s(r,t)dr \]

or

\[ b(t) = \int L(r)s(r,t)dr \]

Problem of source localization:

Estimate \( s(r,t) \) from the measurement \( b(t) \)
What is adaptive beamformer?
Spatial filter

\[ \hat{s}(r, t) = w^T(r)b(t) = [w_1(r), \ldots, w_M(r)]\begin{bmatrix} b_1(t) \\ \vdots \\ b_M(t) \end{bmatrix} = \sum_{m=1}^{M} w_m(r)b_m(t) \]

\[ \uparrow \text{ estimate of } s(r, t) \quad \uparrow \text{ weight vector} \]
Non-adaptive weight

\( w(r) \) is data independent

Adaptive weight

\( w(r) \) is data dependent
Non-adaptive weight

minimum-norm estimate (Hamalainen et al.)

The weight $\mathbf{w}(\mathbf{r})$ is obtained by

$$\mathbf{w}^T(\mathbf{r}) = \mathbf{L}^T(\mathbf{r})\mathbf{G}^{-1}, \text{ where } G_{i,j} = \int l_i(\mathbf{r})l_j^T(\mathbf{r})d\mathbf{r}$$

Inverse solution: $\hat{\mathbf{s}}(\mathbf{r}) = \mathbf{L}^T(\mathbf{r})\mathbf{G}^{-1}\mathbf{b}$

This is erroneous
Property of $G$ matrix

$$G_{i,j} = \int l_i(r) l_j(r) dr$$

Biomagnetic instruments

Overlaps of sensor lead fields is large

$G$ is poorly conditioned

X-ray computed tomography

$G \approx$ unit matrix
$G$ is poorly conditioned

• Apply regularization when calculating $G$

  use $(G + \gamma I)^{-1}$, instead of $G^{-1}$

Bayesian methods

• Do not use $G$

  Adaptive beamforming technique
Adaptive beamformer

**minimum-variance beamformer**

subject to \( \mathbf{w}^T \mathbf{l}(\mathbf{r}, \eta) = \mathbf{L}(\mathbf{r}) \eta = 1 \)

\[
\min_{\mathbf{w}} \mathbf{w}^T \mathbf{D} \mathbf{w} \text{ subject to } \mathbf{w}^T \mathbf{l}(\mathbf{r}, \eta) = \mathbf{L}(\mathbf{r}) \eta = 1
\]

\[
\mathbf{w}^T(\mathbf{r}) = \frac{\mathbf{l}^T(\mathbf{r}) \mathbf{D}^{-1}}{\mathbf{l}^T(\mathbf{r}) \mathbf{D}^{-1} \mathbf{l}(\mathbf{r})}
\]

and

\[
\hat{\mathbf{s}}(\mathbf{r}, t) = \mathbf{w}^T(\mathbf{r}) \mathbf{b}(t) = \frac{\mathbf{l}^T(\mathbf{r}) \mathbf{D}^{-1} \mathbf{b}(t)}{\mathbf{l}^T(\mathbf{r}) \mathbf{D}^{-1} \mathbf{l}(\mathbf{r})}
\]

use \( \mathbf{l}(\mathbf{r}) \) instead of \( \mathbf{l}(\mathbf{r}, \eta) \) for simplicity
Adaptive beamformer

• Spatial resolution can exceed the limit imposed by the sensor-array configuration.

Possibility of providing high spatial resolution

• Strong temporal correlation among source activities degrades the quality of final reconstruction results.
Reconstruction from left-hemisphere data only
Reconstruction from right-hemisphere data only
Right auditory cortex activation

Left auditory cortex activation

correlation coefficient: 0.97
Reconstruction from all-channel data
How does the adaptive beamformer technique perform when sources are moderately correlated?
Influence of the source correlation

Adaptive beamformer cannot perfectly block the signal from correlated sources.

↓

Signal cancellation: intensity of reconstructed source moment decreases

Erroneous time course estimate: reconstructed source time course becomes a mixture of time courses of correlated source.

Spatial blur: spatial resolution is degraded due to the source correlation.
Basic relationship:

\[ \mathbf{w}^T (\mathbf{r}_p) \mathbf{l}(\mathbf{r}_q) = \frac{[\mathbf{R}_s^{-1}]_{pq}}{[\mathbf{R}_s^{-1}]_{pp}} \]


Assume that \( Q \) sources are correlated,

\[ \bar{s}(\mathbf{r}_p, t) = s(\mathbf{r}_p, t) + \sum_{q=1}^{Q} \frac{[\mathbf{R}_s^{-1}]_{pq}}{[\mathbf{R}_s^{-1}]_{pp}} s(\mathbf{r}_q, t) \]

\( \mathbf{R}_s \) : source covariance matrix, \([\mathbf{R}_s^{-1}]_{pq}\) : the \((p,q)\) element of \( \mathbf{R}_s^{-1} \)
When two sources are correlated

$$R_s^{-1} = \frac{1}{\alpha_1^2 \alpha_2^2 (1-\mu^2)} \begin{bmatrix} \alpha_2^2 & -\mu \alpha_1 \alpha_2 \\ -\mu \alpha_1 \alpha_2 & \alpha_1^2 \end{bmatrix}$$

submatrix relating to the correlated two sources

Then

$$\tilde{s}(r_1,t) = s(r_1,t) - \left( \frac{\alpha_1 \mu}{\alpha_2} \right) s(r_2,t)$$

$$\tilde{s}(r_2,t) = -\left( \frac{\alpha_2 \mu}{\alpha_1} \right) s(r_1,t) + s(r_2,t)$$

\(\alpha_j^2\): the jth source power defined by \(\alpha_j^2 = \langle s(r_j,t)^2 \rangle\),

\(\mu\): correlation between the two sources defined by

$$\mu = \frac{\langle s(r_1,t) s(r_2,t) \rangle}{\sqrt{\langle s(r_1,t)^2 \rangle \langle s(r_2,t)^2 \rangle}}$$
Interesting results

\[
\tilde{\mu} = \frac{\left\langle \tilde{s}(\mathbf{r}_1, t)\tilde{s}(\mathbf{r}_2, t) \right\rangle}{\sqrt{\left\langle \tilde{s}(\mathbf{r}_1, t)^2 \right\rangle \left\langle \tilde{s}(\mathbf{r}_2, t)^2 \right\rangle}}
\]

\[\downarrow\]

\[
\tilde{\mu} = \frac{|\alpha_1 \alpha_2 (\mu^3 - \mu)|}{\sqrt{\alpha_1^2 (1 - \mu^2) \alpha_2^2 (1 - \mu^2)}} = |\mu|
\]

Magnitude correlation coefficient calculated using the beamformer outputs is equal to the true magnitude correlation coefficient.
Signal cancellation (when two sources are correlated)

\[
\tilde{s}(\mathbf{r}_1, t) = s(\mathbf{r}_1, t) - \left(\frac{\alpha_1 \mu}{\alpha_2}\right) s(\mathbf{r}_2, t)
\]

\[
\tilde{s}(\mathbf{r}_2, t) = -\left(\frac{\alpha_2 \mu}{\alpha_1}\right) s(\mathbf{r}_1, t) + s(\mathbf{r}_2, t)
\]

\[
\langle \tilde{s}(\mathbf{r}_1, t)^2 \rangle = \alpha_1^2 (1 - \mu^2) = (1 - \mu^2) \langle s(\mathbf{r}_1, t)^2 \rangle
\]

\[
\langle \tilde{s}(\mathbf{r}_2, t)^2 \rangle = \alpha_2^2 (1 - \mu^2) = (1 - \mu^2) \langle s(\mathbf{r}_2, t)^2 \rangle
\]

Source intensity decreases by a factor of \((1 - \mu^2)\)
Intensity vs. correlation

Theoretical curve: \( \propto \sqrt{1-\mu^2} \)
37-channel sensor array

source correlation: zero

source correlation: 0.8

SNR=8
Reconstruction results

$\mu = 0$  $\mu = 0.5$  $\mu = 0.6$

$\mu = 0.7$  $\mu = 0.8$  $\mu = 0.95$
\[
\tilde{s}(\mathbf{r}_1, t) = s(\mathbf{r}_1, t) - \left( \frac{\alpha_1 \mu}{\alpha_2} \right) s(\mathbf{r}_2, t)
\]

\[
\tilde{s}(\mathbf{r}_2, t) = -\left( \frac{\alpha_2 \mu}{\alpha_1} \right) s(\mathbf{r}_1, t) + s(\mathbf{r}_2, t)
\]
Time course retrieval

Two-source correlation cases

\[
\begin{bmatrix}
\hat{s}(r_1, t) \\
\hat{s}(r_2, t)
\end{bmatrix} =
\begin{bmatrix}
1 & -(\alpha_1 / \alpha_2) \mu \\
-(\alpha_2 / \alpha_1) \mu & 1
\end{bmatrix}
\begin{bmatrix}
s(r_1, t) \\
s(r_2, t)
\end{bmatrix}
\]

\[
\begin{bmatrix}
\hat{s}(r_1, t) \\
\hat{s}(r_2, t)
\end{bmatrix} =
\begin{bmatrix}
1 & -(\alpha_1 / \alpha_2) \mu \\
-(\alpha_2 / \alpha_1) \mu & 1
\end{bmatrix}^{-1}
\begin{bmatrix}
\hat{s}(r_1, t) \\
\hat{s}(r_2, t)
\end{bmatrix}
\]

\[
\hat{\mu} = \tilde{\mu}
\]
\[
\hat{\alpha}_j^2 = \alpha_j^2 / (1 - \tilde{\mu}^2)
\]
Time course retrieval experiments for two correlated sources

original

beamformer output

retrieved
Influence of the source correlation on the spatial resolution
37-channel sensor array

\( \mu = 0.1 \)

\( \mu = 0.5 \)

\( \mu = 0.8 \)
Assume two sources with identical power

\( \mathbf{r}_1, \mathbf{r}_2 : \) source locations, \( \eta_1, \eta_2 : \) source orientations

\( \alpha^2 : \) power of each source, \( \mu : \) correlation coefficient

Define \( \mathbf{l}_1 = \mathbf{L}(\mathbf{r}_1)\eta_1 \) and \( \mathbf{l}_2 = \mathbf{L}(\mathbf{r}_2)\eta_2 \)

Then,
\[
\mathbf{D} = \sigma^2 \mathbf{I} + \alpha^2 \begin{bmatrix} \mathbf{l}_1 & \mathbf{l}_2 \end{bmatrix} \begin{bmatrix} 1 & \mu \\ \mu & 1 \end{bmatrix} \begin{bmatrix} \mathbf{l}_1 & \mathbf{l}_2 \end{bmatrix}^T
\]

\[
= \sigma^2 (\mathbf{I} + \frac{\alpha^2}{\sigma^2} \mathbf{l}_1\mathbf{l}_1^T + \mathbf{l}_2\mathbf{l}_2^T + \mu (\mathbf{l}_1\mathbf{l}_2^T + \mathbf{l}_2\mathbf{l}_1^T ))
\]

beamformer response at \( \mathbf{r} \)

\[
p(\mathbf{r}) = \sum_{j=x,y,z} w_j^T(\mathbf{r})\mathbf{D}w_j(\mathbf{r})
\]
Lorenzean curve fitting

Lorenzean curve

\[ f(y) = \frac{1}{1 + [(y - y_j)/\Delta]^2} \]

\( \Delta \): FWHM of the curve
Summary

The performance degradation of the adaptive beamformer techniques in the presence of correlated sources was analyzed when two correlated sources exist:

• The performance is generally not significantly degraded in the presence of moderately correlated sources ($\mu < 0.7$).

• The time course estimate may be erroneous even for such moderate degree of source correlation.

• A method is developed for retrieving the original time courses, when the number of correlated sources are two or three and this number is known.